Sample Question Paper - 10
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

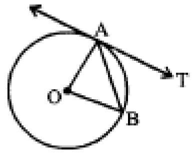
1. Find the 12^{th} term from the end of the arithmetic progression 1,4,7,10,..., 88. **[2]**

OR

Determine the A.P. whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

2. Solve the given quadratic equation by factorization $y^2 - 3 = 0$ **[2]**

3. In the adjoining figure, O is the centre of the circle, AB is a chord and AT is tangent at A. If $\angle AOB$ = 100°, then find $\angle BAT$. **[2]**



4. If the heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4, what is the ratio of their volumes? **[2]**

5. i. Find the mode of the following data: 25, 16, 19, 48, 19, 20, 34, 15, 19, 20, 21, 24, 19,16, 22, 16, 18, 20, 16, 19. **[2]**
   ii. If one of the 19’s is changed to 16 in the above data, find the new mode.

6. In a rectangular part of dimensions, 50 m $\times$ 40 m a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be 1184 m^2. Find the length and breadth of the pond. **[2]**

OR

Solve the quadratic equation by factorization:

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Section B

7. Find the mean marks from the following data: **[3]**

Marks	**Number of students**
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

8. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. **[3]**

9. Compute the median for the following cumulative frequency distribution: **[3]**

Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90	Less than 100
0	4	16	30	46	66	82	92	100

10. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° respectively. Find the height of the balloon above the ground. **[3]**

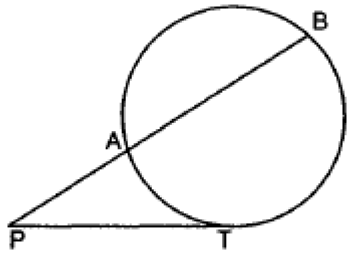
OR

A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

Section C

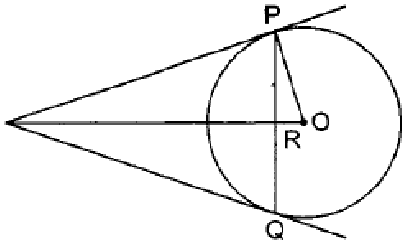
11. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm; find the total surface area and volume of the toy. **[4]**

12. In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT. **[4]**



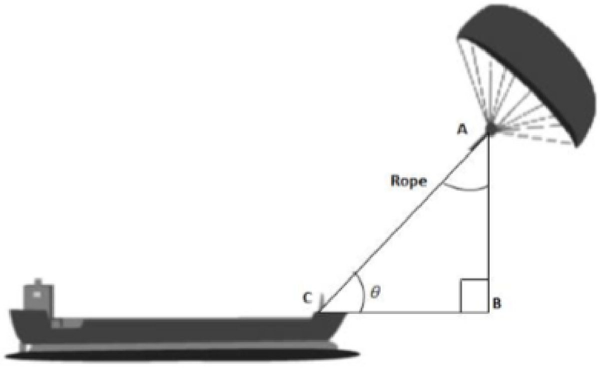
OR

In figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



13. Marine Insight uses extensive utilization of wind energy to move a vessel in the seawater. They allow the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively. **[4]**

Based on the following figure, answer the question that follows:



i. In the given figure, if $\sin \theta = \cos(3\theta - 30^\circ)$, where θ and $3\theta - 30^\circ$ are acute angles, then find the value of θ.
ii. What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

14. Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily. **[4]**

i. If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it
ii. Find the total money he saved.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. Let the total number of terms be n.
   So,
   First term (a) = 1
   Last term (a_n) = 88
   Common difference, d= 4-1=3
   Now, as we know,
   a_n=a+(n-1)d
   So, for the last term,
   88=1+(n-1)3
   88=1+3n-3
   88=-2+3n
   88+2=3n
   Further simplifying,
   90=3n
   n=$\frac{90}{3}$
   n=30
   So, the 12^{th} term from the end means the 19^{th} term from the beginning.So, for the 19^{th} term (n = 19)
   a_{19}=1+(19-1)3
   =1+(18)3
   =1+54
   =55
   Therefore, the 12^{th} term from the end of the given A.P. is 55.

   OR

   Let the given AP be a,a+d,a+2d,a+3d,......
   4^{th} term = 18 (given)
   a+3d=18 ....(1)
   Difference between 15^{th} term and 9^{th} term =30 (given)
   (a + 14d) - (a + 8d) = 30
   a + 14d - a -8d = 30
   6d = 30
   d = $\frac{30}{6}$
   d = 5
   put d=5 in eq. (1),we get,
   a + 3 $\times$5=18
   a + 15 = 18
   a = 18 - 15
   a = 3
   A.P. is a, a + d, a + 2d,....
   i.e 3, 3 + 5, 3 + 10
   i.e 3, 8, 13.
2. y^2 - 3 = 0
   $\implies y^2-(\sqrt{3})^2=0$
   $\implies (y+\sqrt{3})(y-\sqrt{3})=0$ (Using Identity $a^2 - b^2$= (a + b)(a – b))
   Either $y+\sqrt{3}=0\, or\, y-\sqrt{3}=0$
   $\therefore y=-\sqrt{3},\sqrt{3}$
   Thus, $y=-\sqrt{3}\, and\, \sqrt{3}$ are the roots of the given equation.

3. From the given figure AO and BO are the radius of the circle.
So, AO = BO (both are radii)
Hence, in Δ AOB: $\angle OBA = \angle OAB$ (angles opposite to equal sides are equal)......(1)
Also given , $\angle AOB = 100^\circ$ .....(2)
In △AOB, We have
$\angle AOB + \angle OBA + \angle OAB = 180^\circ$(sum of angles in a triangle is 180$^{\text{o}}$)
$\Rightarrow$ 100° + $\angle OAB + \angle OAB$ = 180° [from(1) & (2)]
$\Rightarrow 2\angle OAB = 80^\circ$
$\Rightarrow \angle OAB = 40^\circ$
We know that the radius and tangent at their point of contact are perpendicular to each other
$\therefore \angle OAT = 90^\circ$
$\Rightarrow \angle OAB + \angle BAT = 90^\circ$
$\Rightarrow \angle BAT = 90^\circ - 40^\circ = 50^\circ$

4. Ratio in the heights of two cones = 1 : 2 and ratio in the perimeter of their bases = 3 : 4
Let r_1, r_2 be the radii of two cones and h_1 and h_2 be their heights
$\therefore \frac{h_1}{h_2} = \frac{1}{2}$
and $\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$
$\Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$
Now $\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$
$= \frac{r_1^2 h_1}{r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$
$= \left(\frac{3}{4}\right)^2 \times \frac{1}{2} = \frac{9}{16} \times \frac{1}{2} = \frac{9}{32}$
∴ Ratio in their volumes = 9 : 32

5. The given data is 25, 16, 19, 48, 19, 20, 34, 15, 19, 20, 21, 24, 19,16, 22, 16, 18, 20, 16, 19.
   i. mode is 19
   ii. If 19 is changed to 16, then frequency of 16 is 5 and frequency of 19 becomes 4 i.e., 16 has maximum frequency. Therefore, 16 is the new mode.

6. Let width of grass strip be x mts.
∴ Length of pond = (50 - 2x) mt and Breadth of pond = (40 - 2x) mt
And area of park - area of pond = area of grass strip
$\Rightarrow (50 \times 40) - (50 - 2x)(40 - 2x) = 1184$
$\Rightarrow \quad 2000 - 2000 + 180x - 4x^2 = 1184$
$\Rightarrow \quad x^2 - 45x + 296 = 0$
$\Rightarrow \quad x^2 - 37x - 8x + 296 = 0$
$\Rightarrow$ x(x - 37) - 8(x - 37) = 0
or, (x - 37) = 0 or (x - 8) = 0
$\Rightarrow$ x = 8, 37
37 is rejected
Length of pond = $50 - 2 \times 8$ = 50 - 16 = 34m and breadth of pond 40 - 2(8) = 40 - 16 =24 m.

OR

Given, $a^2b^2x^2 + b^2x - a^2x - 1 = 0$
or, $b^2x\left(a^2x + 1\right) - 1\left(a^2x + 1\right) = 0$
or, $\left(b^2x - 1\right)\left(a^2x + 1\right) = 0$
$\left(b^2x - 1\right) = 0$ or $\left(a^2x + 1\right) = 0$
$b^2x = 1$ or $a^2x = -1$
$\therefore \quad x = \frac{1}{b^2}$, or, $x = -\frac{1}{a^2}$
Hence, roots = $\frac{1}{b^2}, -\frac{1}{a^2}$

Section B

7. Calculation of Mean :

C.I.	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0 - 10	5	5	-4	-20
10 - 20	4	15	-3	-12
20 - 30	8	25	-2	-16
30 - 40	12	35	-1	-12
40 - 50	16	45	0	0
50 - 60	15	55	1	15
60 - 70	10	65	2	20
70 - 80	8	75	3	24
80 - 90	5	85	4	20
90 - 100	2	95	5	10
Total	$\Sigma f_i = 85$			$\Sigma f_i u_i = 29$

a = 45, h = 10

$\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 45 + \frac{29}{85} \times 10$

$= 45 + 3.417 = 48.41$

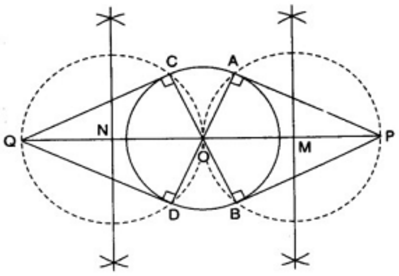
Therefore, the mean marks are 48.41.

8. Required: To draw a circle of radius 3 cm and take two points P and Q on one of its extended its centre and then draw tangent o the circle from these two point P and Q.

   Steps of construction:

   i. Bisect PO. Let M be the mid-point of PO.
   ii. Taking M as centre and MO as radius, draw a circle. Let it intersect the Given circle at the point A and B.
   iii. Join PA and PB

      Then, PA and PB are the required two tangents.



   iv. Bisect QO. Let N be the mid-point of QO.
   v. Taking N as centre and NO as radius, draw a circle. Let it intersect the given circle at the point C and D.
   vi. Join QC and QD

      Then QC and QD

   Justification: Join OA and OB

   Then ∠ PAO is an Angle in the semicircle and, therefore,

   ∠ PAO = 90°

   ⇒ PA ⊥ OA

   Since OA is a radius of given circle, PA has to be tangent to the circle.

   Similarly, PB is also a tangent to the circle.

   Again, Join OC and OD.

   Then ∠ QCO is an angle on the semicircle and, therefore ∠ QOC = 90°

   Since OC is a radius of the given circle, QC has to be a tangent to the circle.

   Similarly, QD is also a tangent to the circle.

9. We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median

Class intervals	**Frequency (f)**	**Cumulative frequency (c.f.)**
20-30	4	4
30-40	12	16
40-50	14	30
50-60	16	46
60-70	20	66
70-80	16	82
80-90	10	92
90-100	8	100
		N = Σf_i = **100**

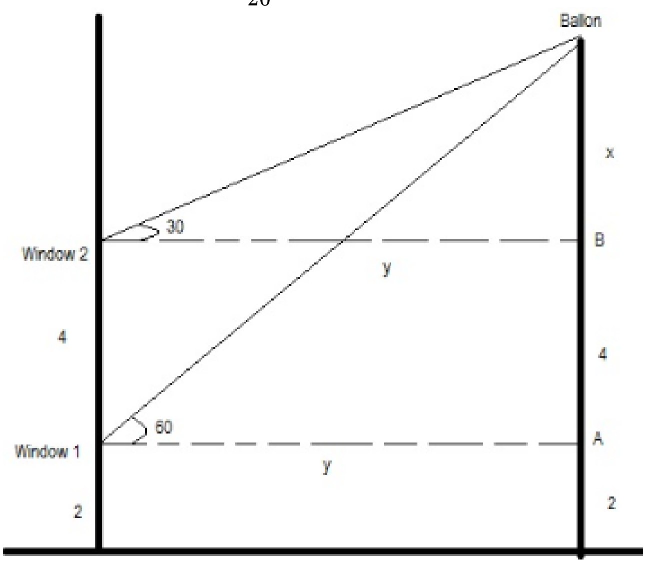
Here, N = Σf_i = 100 $\therefore$ $\frac{N}{2}$ = 50

We observe that the cumulative frequency just greater than $\frac{N}{2}$ = 50 is 66 and the corresponding class is 60-70. So, 60-70 is the median class.

$\therefore$ l = 60, f = 20, F = 46 and h = 10

Now, Median = l + $\frac{\frac{N}{2}-F}{f}\times$ h

$\Rightarrow$ Median = 60 + $\frac{50-46}{20}\times$ 10 = 62

10. 

From the figure,

let the height of balloon is = x+4+2 = x+6

$\tan 30^\circ = \frac{x}{y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

$\Rightarrow y = \sqrt{3}x$ .... (i)

And, $\tan 60^\circ = \frac{x+4}{y}$

$\Rightarrow \tan 60^\circ = \frac{x+4}{y}$

$\Rightarrow \sqrt{3} = \frac{x+4}{y}$

$\Rightarrow \sqrt{3} = \frac{x+4}{\sqrt{3}x}$

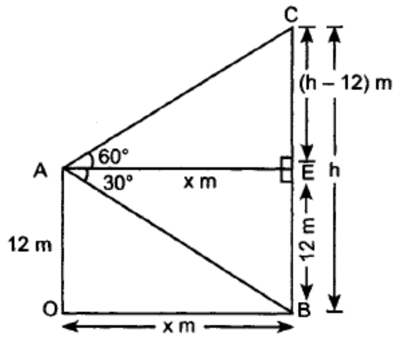
$\Rightarrow 3x = x + 4$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

Thus, height of balloon $= x + 4 + 2 \Rightarrow 2 + 4 + 2 = 8m$

OR

A is the position of the man, OA = 12 m, BC is cliff.



BC = h m and CE = (h - 12)m

Let $\mathrm{AE} = \mathrm{OB} = x$ m

In right angled triangle AEB,

$\frac{\mathrm{AE}}{\mathrm{BE}} = \cot 30^\circ \Rightarrow \mathrm{AE} = 12 \times \sqrt{3}$

$= 12 \times 1.732\mathrm{m} = 20.78\mathrm{m}$

∴ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC,

$\frac{CE}{AE} = \tan 60^\circ \Rightarrow \frac{h-12}{12\sqrt{3}} = \sqrt{3}$

$h - 12 = 36 \Rightarrow h = 48\mathrm{m}$

Section C

11. The Radius of the toy (r) = 3.5 cm

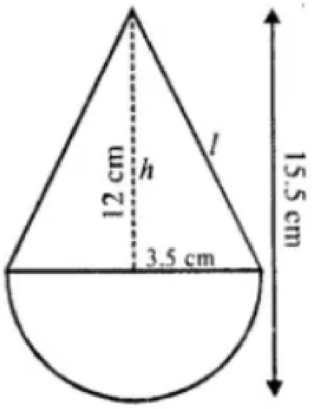
Total height of the toy = 15.5 cm

∴ Height of the conical part is = 15.5 - 3.5 = 12 cm

Slant height of the conical part (l)

$= \sqrt{r^2 + h^2} = \sqrt{(3.5)^2 + (12)^2}$

$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\mathrm{cm}$



i. Now total surface area of the toy = curved surface area of conical part + curved surface area of hemipherical part

$= \pi r l + 2\pi r^2 = \pi r(l + 2r)$

$= \frac{22}{7} \times 3.5(12.5 + 2 \times 3.5)\ \mathrm{cm}^2$

= 11 (12.5 + 7) = 11 × 19.5 cm^2

= 214.5 cm^2

ii. Volume of the toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7}(3.5)^2(12 + 2 \times 3.5)\mathrm{cm}^3$

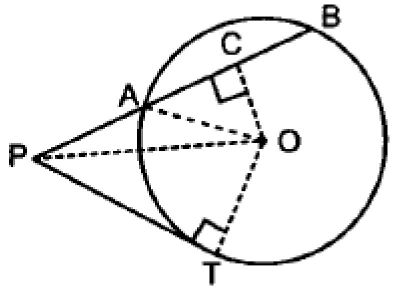
$= \frac{1}{3} \times \frac{22}{7} \times 12.25(12 + 7)\mathrm{cm}^3$

$= \frac{22}{3} \times 1.75 \times 19\mathrm{cm}^3$

$= \frac{731.5}{3}$ = 243.83 cm^3

12. Given, PT is tangent to the circle at T.

PA = 4 cm and AB = 5 cm.



Construction: Draw OC $\perp$ AB and join OP, OT and OA.

Proof: In right $\triangle$OCP

$OP^2 = PC^2 + OC^2$

$OP^2 = [AP + AC]^2 + OC^2$

$\mathbf{OP}^2 = \left[4 + \frac{1}{2}\mathbf{AB}\right]^2 + \mathbf{OC}^2$ [OC $\perp$ AB, AC = BC]

$\Rightarrow OP^2 = \left(4 + \frac{5}{2}\right)^2 + OC^2$

$\Rightarrow OP^2 = \left(\frac{13}{2}\right)^2 + OC^2$ ..(i)

In right $\triangle$OCA,

$OA^2 = OC^2 + AC^2$

$OA^2 - AC^2 = OC^2$

$OA^2 - \left(\frac{5}{2}\right)^2 = OC^2$ ..,(ii)

$\therefore$ eq (i) becomes.

$OP^2 = \left(\frac{13}{2}\right)^2 + OA^2 - \left(\frac{5}{2}\right)^2$

$OP^2 = \frac{169}{4} - \frac{25}{4} + OA^2$

$OP^2 = \frac{144}{4} + OA^2$

$\Rightarrow OP^2 = 36 + OA^2$ ...(iii)

Also, $OP^2 = OT^2 + PT^2$ ...(iv)

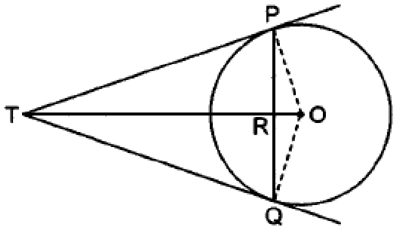
from (iv) and (iii),

$PT^2 + OT^2 = 36 + OA^2$

$\Rightarrow PT^2 = 36$ [$\therefore$ OT = OA = radii]

$PT = 6cm.$

OR



Given, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T.

Construction: Join OP and OQ.

In triangles OTP and OTQ, OT is common

$OP = OQ$(radii)

$TP = TQ$

$\therefore \Delta OPT \cong \Delta OQT$

$\therefore \angle POT = \angle QOT$

Consider, triangles OPR and OQR;

$OP = OQ$ (radii); OP is common

$\angle POR = \angle QOR$ [from(i)]

$\therefore \Delta OPR \cong \Delta OQR$ (SAS cong. rule)

$\therefore PR = RQ = \frac{1}{2} \times 16 = 8cm...(ii)$

$\angle ORP = \angle ORQ = 90^\circ$ ....(iii)

In right angled triangle TRP,

$TR^2 = TP^2 - (8)^2 = TP^2 - 64$ [from (iii)] ,..(iv)]

Also $OT^2 = TP^2 + (10)^2$

$(TR + 6)^2 = TP^2 + 100[\because OR = \sqrt{100 - 64} = 6]$

$TR^2 + 12TR + 36 = TP^2 + 100$

$TP^2 - 64 + 12TR + 36 = TP^2 + 100$

$12TR = 128 \Rightarrow TR = \frac{32}{3}cm$

From (iv)

$\left(\frac{32}{3}\right)^2 = TP^2 - 64$

$\Rightarrow TP^2 = \frac{1024}{9} + 64 = \frac{1024+576}{9} = \frac{1600}{9}$

$\Rightarrow \quad TP = \frac{40}{3}cm$

13. i. sin θ = cos(3θ - 30^o)

cos(90^o - θ) = cos(3θ - 30^o)

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$

ii. $\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o}$ = $\frac{150}{\frac{1}{2}}$ = 150$\times$2 = 300 m

14. Child's savings day wise are,

$\underbrace{5}_{1\ coin}, \underbrace{10}_{2\ coins}, \underbrace{15}_{3\ coins}, \underbrace{20}_{4\ coins}, \underbrace{25}_{5\ coins}$ .... to $\underbrace{n\ days}_{n\ coins}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1] = 190$

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow$ n^2 + n - 380 = 0

$\Rightarrow$ (n + 20)(n−19) = 0$\Rightarrow$(n + 20)(n - 19) = 0

$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

= $\frac{19}{2}$[2 $\times$ 5 + (19 - 1)5]

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

So, number of days = 19

and total money she shaved = Rs. 950